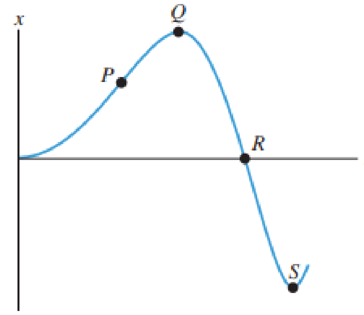
All conditions are assumed ideal unless mentioned in the question.

A. Multiple choice questions:

1) At which of the points in the position-time graph alongside is the speed of the particle maximum?

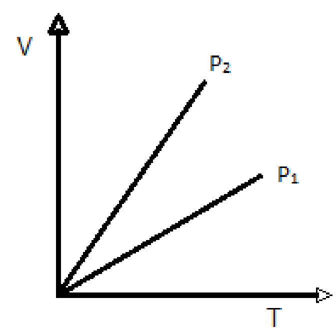


   a. P c. R
   b. Q d. S

2) P is a point on a wheel of radius R rolling in a horizontal road. Initially, the point P is in contact with ground. The wheel rolls through half of the revolution. What is the displacement of the point P?

   a. πR c. $(\sqrt{\pi^2+1})$ R
   b. 2πR d. $(\sqrt{\pi^2+4})$ R

3) In the following V-T diagram, what is the relation between P_1 and P_2?

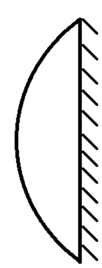


   a. $P_1 = P_2$
   b. $P_1 > P_2$
   c. $P_1 < P_2$
   d. Cannot be predicted

4) An object is moving towards a plane mirror with a velocity v making a certain angle θ with the normal of the plane mirror. The velocity of the image with respect to the object is

   a. vcosθ
   b. vsinθ
   c. 2vcosθ
   d. 2vsinθ

5) The plane face of the Plano-convex lens of focal length 20 cm is silvered as shown in figure alongside. What type of mirror will it behave and of what focal length (f)?



   a. Concave, f = 10 cm
   b. Convex, f = 10 cm
   c. Concave, f = 20 cm

d. Convex, f = 20 cm

6) The attraction force between two equal charges is F. If half of the charge from the first charge is transferred to the second one, what will be the new attraction force in terms of the old force?

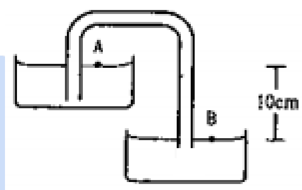
a. F

b. $\frac{1}{2}$ F

c. 2 F

d. $\frac{3}{4}$ F

7) Figure alongside shows a siphon. The liquid shown is water. The pressure difference $P_B - P_A$ between the points A and B is



a. 400 Pa

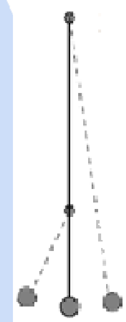
b. 1000 Pa

c. 3000 Pa

d. zero

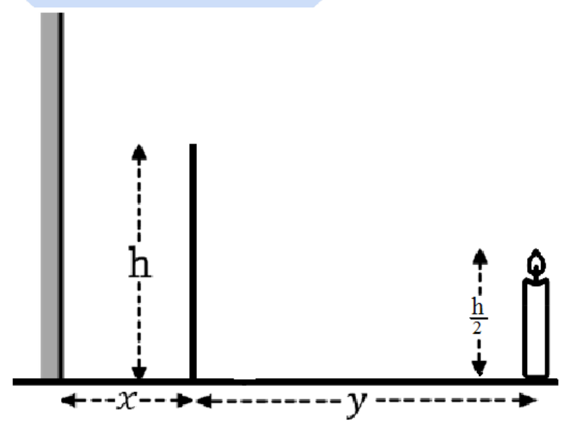
B. Short question answers:

1) A simple pendulum of length L is constructed from a point object of mass m suspended by a massless string attached to a fixed pivot point. A small peg is placed a distance 2L/3 directly below the fixed pivot point so that the pendulum would swing as shown in the figure below. The mass is displaced 5 degrees from the vertical and released. How long does it take to return to its starting position?

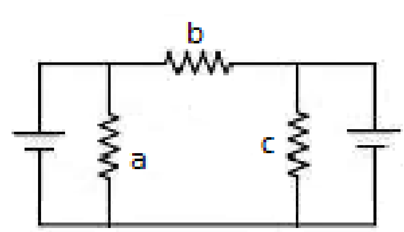


2) 1 kg of ice at 0˚C is mixed with 1 kg of steam at 100˚C. What will be the composition and temperature of the system when thermal equilibrium is reached?

3) A stick and a candle have lengths of h and $h/_2$ respectively. The candle melts completely in time t at a steady rate. The distances x and y are as shown in the figure. What is the speed of the shadow on the screen?

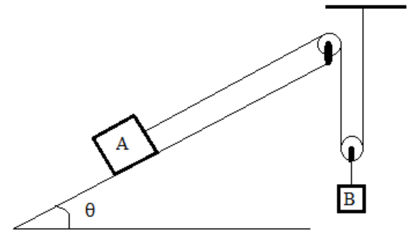


4) In the circuit alongside, the cells are ideal with an EMF of 10 V each. Each resistor has a resistance of 10 Ω. Find the current flowing through each resistor.



C. Numerical:

1) Block A of mass M is kept on an inclined plane of inclination of $\theta=30^\circ$. The coefficient of friction between the block and the inclined plane is μ. Block A is connected to block B of unknown mass by two pulleys as shown in figure alongside.



   a. What is the maximum value of the mass of B in terms of M so that the blocks remain stationary?
   b. If the mass of B is 2M, find the minimum value of μ so that the blocks remain stationary.
   c. What would be the acceleration of block A if $\mu= 0.1$?

2) The relation between the orbital time period T and the semi-major axis a of an elliptical orbit of a planet is found to be

$$T = pa^q$$

   where, p and q are constants.

   a. Linearize the equation to obtain a straight line on a graph of T vs. a.
   b. The table below shows the data for the five nearest planets to the sun. Process the data to obtain a straight line graph. (Note: the distance is in Astronomical Units. 1 AU = 1.5×10^{11}m.)

a/AU	0.39	0.72	1.00	1.53	5.20
T/year	0.24	0.62	1.00	1.88	11.86

   c. Plot the processed data and fit to a straight line.
   d. Find the value of p and q from the graph.

   If you did it correctly, you will obtain the same relation that German mathematician and astronomer Johannes Kepler obtained using the data of planetary motion collected by Danish astronomer Tycho Brahe in back in 16^{th} century. This law is popularly known as Kepler's third law of Planetary motion.

3) A vertical cylinder of cross-sectional area S contains n moles of an ideal monatomic gas under a piston of mass m. At a certain instant, a heater which transmits an amount of heat q per unit time to the gas is switched on. Determine the established velocity v of the piston under the condition that the gas pressure under the piston is constant, and the gas under the piston is thermally insulated.

   A person is claiming that such a system can be economically used to lift a car instead of a hydraulic lift with an efficiency of 60%. If the car and the piston weighs

2000kg altogether, the area of the piston is $5m^2$ and the atmospheric pressure is 1×10^5 Pa, calculate the efficiency of the lift. How practical would this system be?

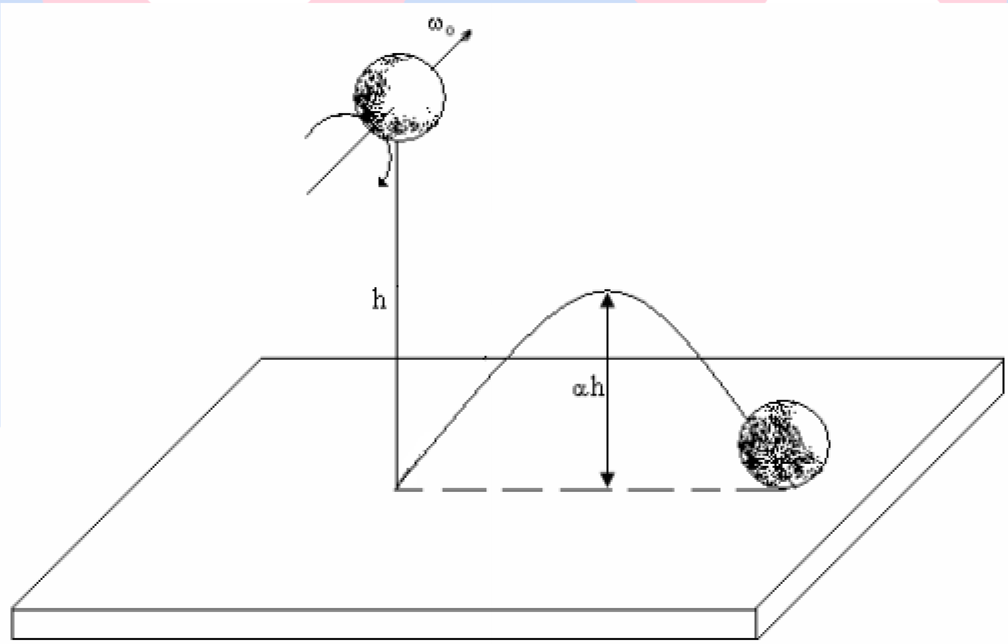
4) Two small, equally charged spheres, each of mass m, are suspended from the same point by silk threads of length L. Initially, the spheres are separated by distance $x << L$. As the charge leaks out at the rate dq/dt, the spheres approach each other with relative velocity $v = a/\sqrt{x}$, where a is a constant. Show that the rate at which charge leaks out is:

$$\frac{dq}{dt} = \frac{3}{2}\,a\sqrt{\frac{2\pi\epsilon_o mg}{L}}$$

Calculate the value of a if the charge leaks out at a rate of 0.01 μC/s, m=50 g, and L=5cm.

D. Application problem:

The figure below shows a hollow spherical ball of mass m and radius R. Before falling to the floor its center of mass is at rest, but the ball is spinning with angular velocity w_0 about a horizontal axis through its center. The lowest point of the ball is at a height h above the floor.



When released, the ball falls under gravity, and rebounds to a new height such that its lowest point is now αh above the floor. Ignore the presence of the air.

The acceleration due the gravity is g, the dynamic coefficient of friction between the ball and the floor is μ_k, and the moment of inertia of the ball about the given axis is:

$$I = \frac{2}{3}mR^2$$

A ball is said to slip when the velocity of the instantaneous point of contact is not zero. Consider that the ball slips during the entire impact time.

a. Find the velocity u_0 of the ball at the instant before impact.
b. During the time of impact, the force due to gravity is negligible compared to the normal reaction. Draw a free body diagram of the ball during impact and write the expressions for the changes in linear and angular momenta in the time of impact.
c. Find the vertical and horizontal velocities of the ball after impact.
d. What is the horizontal distance traveled in flight between the first and second impacts?
e. Obtain the minimum value of w_0 for the ball to slip throughout the impact. If this condition was not met, what would be the horizontal and angular velocities of the ball after impact?

Now, we will discuss the physics of topspin shot, a widely used technique in table tennis in which a player hits the ball so that the ball rotates as shown in the following figure.

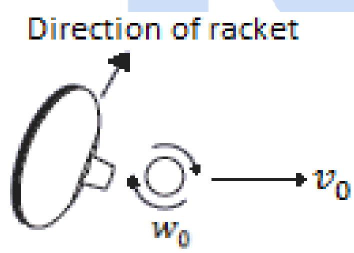


Fig: Topspin shot

Except that the ball has an initial horizontal velocity of v_0, the situation here is same as the one described before where the ball slips throughout the impact.

f. One characteristic of topspin shot is that the ball moves with a greater horizontal velocity after impact, which also leads to a decrease in the angle it makes with the horizontal as shown in the figure below. Using and modifying the results from previous parts of the problem, find tan β. If h=20 cm, α=0.6, μ_k=0.2, v_0= 10m/s, calculate the value of θ and β.

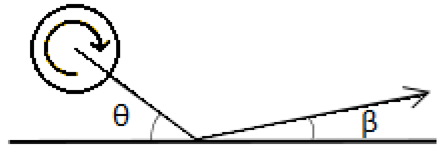


Fig: Angles made by the ball just before and after impact

g. If we don't not neglect the presence of air, the ball falls down quickly in a topspin shot than in a normal shot which enables the players to use powerful shots without crossing the table. Explain why it happens if the force of gravity acting on the ball is the same in both type of shots.

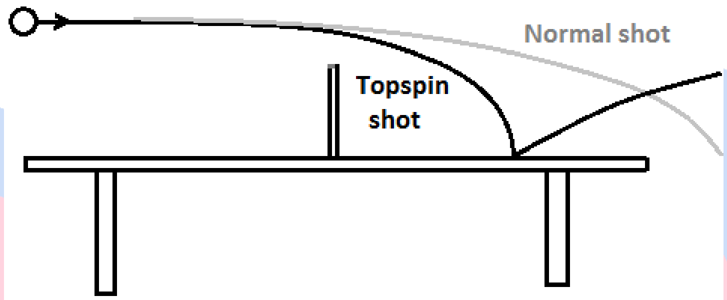


Fig: Trajectory of a ball in topspin shot vs. normal shot