Final selection test F.M : 100

Time : 3 Hrs

Answer all questions **Date: May 26, 2018**

Q1. A cloud consists of mist particles (tiny droplets) suspended in air, uniformly distributed and at rest. Consider a rain drop falling through them. Assume that the rain drop is spherical at all times. Ignore the air resistance on the rain drop.

a) Write down an equation for the change in mass of the rain drop relating to change in its size.

b) Assume that the droplet gets added to the raindrop when the later hits it. Develop a relation for the change in mass of the rain drop as the drop falls through the cloud. Hence obtain expressions for the velocity and the acceleration of the rain drop at any given instant.

c) Use Newton's 2^{nd} law to obtain an equation of motion for the size of the rain drop.

d) Considering that the rain drop falls with constant acceleration at large times, find its acceleration. [10]

Q2 a). A cubical body at rest has each side of 1 m. It moves with a velocity of 0.6c along the x-direction. What is the shape and dimensions of the body seen by an observer at rest? [5]

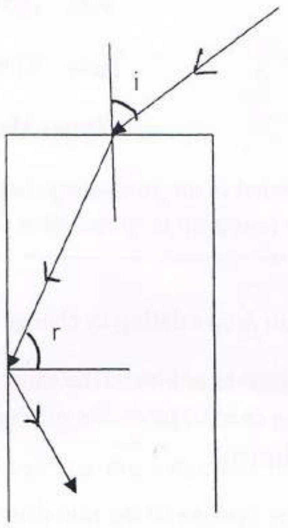
b). A clock is put in a satellite in a geostationary orbit and another clock vertically below it on the earth's surface. You have to find the time difference between the two clocks. It is advisable to work in an inertial frame centered on the earth but not rotating with it.

i) Given the radius of the earth to be 6400 km, find the speed of the clock on the earth.
ii) Find the speed of the clock on the satellite.
iii) Based on two results above, calculate the time lag between the two clocks. [5]

Q3 a). 2.00 mole of an ideal gas undergoes a reversible isothermal compression from V_1 to volume $V_2 = 0.500V_1$ at temperature T= 400K. Find (a) the work done by the gas and (b) the entropy change of the gas. (c) If the compression is reversible and adiabatic instead of isothermal, what is the entropy change of the gas? [5]

b) Find the entropy change of an aluminum bar of mass m=3.0 kg on its heating from temperature $T_1 = 300K$ up to $T_2 = 600K$, if in this temperature interval the specific heat capacity of aluminum varies as $c = a + bT$, where a = 770 J/kg K, b = 0.46 J/kg K. [5]

Q4 a). What is the minimum index of refraction for the plastic rod (Fig. below) which will insure that any ray entering at the end will always be totally reflected in the rod? [5]



b). You have been asked to compare four different proposals for telescope to be placed in orbit, above the blurring effects of the earth's atmosphere. Rank the proposed telescope in order of their ability to resolve small details, from best to worst. i) a radio telescope 100m in diameter observing at a wavelength of 21 cm. ii)an optical telescope 2.0 m in diameter observing at a wavelength of 500 nm; iii) an ultraviolet telescope 1.0 m in diameter observing at a wavelength of 100 nm; iv) an infrared telescope 2.0 m in diameter observing at a wavelength of 1010 μm. [5]

Q5. An air spaced parallel plate capacitor has square plates of side l each separated by a distance d.

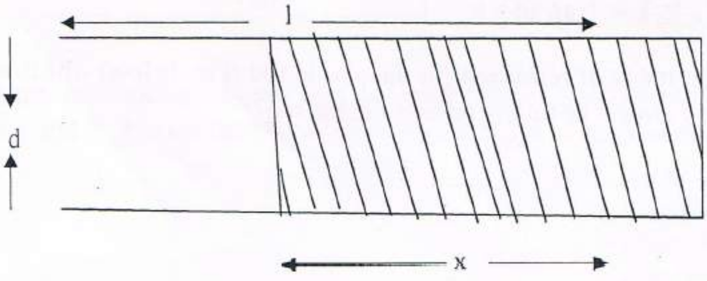
a) Write down an expression for its capacitance C.

b) A square block of dielectric of side l, thickness d and relative permittivity ε_r is now inserted so as to completely fill the space between the plates. Calculate the change in the stored energy of the system if:

i) the plates have a fixed charge Q,

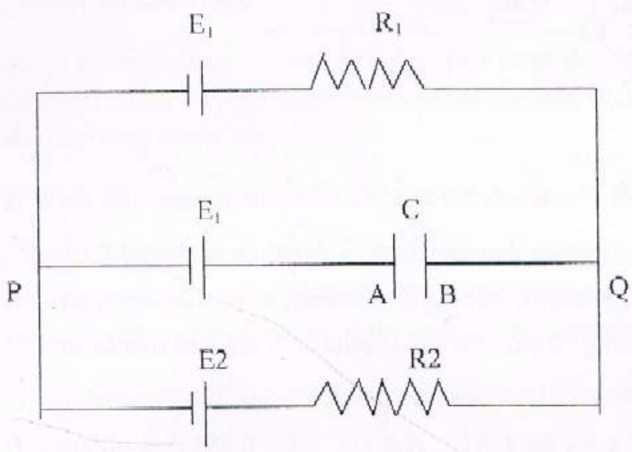
ii) a constant potential difference V is maintained between the plates.

c) With the battery still connected between the plates, the block of dielectric is withdrawn in a direction parallel to one side of the plates until only a length x remains between the plates (see figure). Find the magnitude and direction of the force which acts on the block when it is in this position.

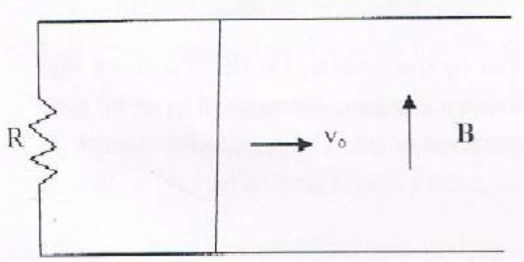


(Assume l>>d and ignore edge effects throughout this problem). [10]

Q6 a). Find the potential difference between the plates A and B of the capacitor C in the circuit shown below. It is given that $E_1 = 1V$, $E_2 = 2.5$ V, $R_1 = 10\ \Omega$ and $R_2 = 20\ \Omega$. Neglect internal resistances of the batteries. [5]



b). A straight conducting wire of mass m can slide without friction along horizontal conducting rails separated by a distance d. The rails are connected to a resistance R and placed in a vertically uniform magnetic field of induction **B**. The conducting wire is pushed initially at a velocity v_0. Determine the distance covered by the wire before it comes to rest. [5]



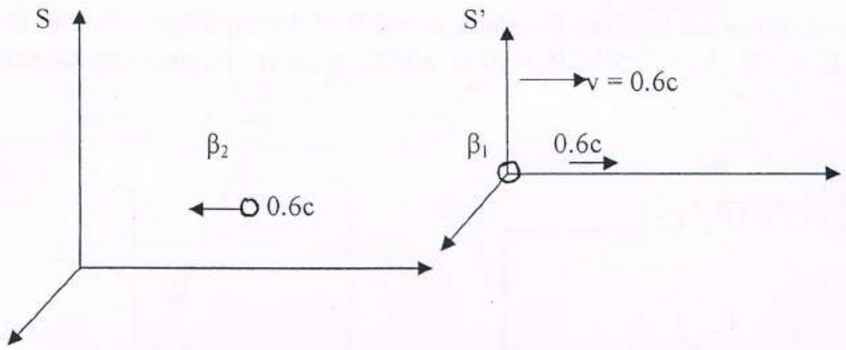
Q7 a). A microscope using photon is employed to locate an electron in an atom to within a distance of 0.2 Å. What is the uncertainty in the velocity of the electron located in this way? [5]

b) Free neutrons have decay constant of $1.10 \times 10^{-3}\ s^{-1}$. If the de Broglie wavelength of the neutron in a parallel beam is 1nm , determine the distance from the source where the beam intensity has dropped to half its starting value. [5]

[$m_n = m_p = 1.67\times 10^{-27}$ kg , $\hbar = 1.05\times 10^{-34}$ Js, $m_e = 9.1\times 10^{-31}$kg, $e = 1.6\times 10^{-19}$C]

Q8 a). Rutherford used alpha particle of energy 7.7 Mev to probe the interior of the atom. What is the de Broglie wavelength of such alpha particle? Is it adequate to treat the motion of these alpha particles by classical mechanics down to distance 10^{-14} m from the nucleus, as Rutherford did? [5]

b) Two β – particles move in opposite direction with velocity 0.6c in the laboratory, S –frame (Fig. below). Calculate the velocity of one β –particle in the moving frame attached to the other β- particle by applying relativistic transformation. Repeat the calculations by taking speed of β-particles as 0.06c. Comment on your results. [5]



Q9. The glider on a horizontal air track is attached to a spring that causes it to oscillate back and forth. The total energy of the system is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, where m is the glider's mass, v is its velocity, k is the spring force constant and x is the extension of spring from the equilibrium. A student makes the following measurements:

$m = 0.230 \pm 0.001\ kg,\quad v = 0.89 \pm 0.01\ m/s \quad k = 1.03 \pm 0.01\ N/m \quad x = 0.551 \pm 0.005$ m

i) What is his best estimate for the total energy? He next measures the position x_{max} of the glider at the extreme end of its oscillation where v=0 as

$$x_{max} = 0.698 \pm 0.002\ m$$

ii) What is his best estimate for the energy at the end point?

iii) Are his results consistent with the principle of conservation of energy? Explain [10]

Q10. The power delivered to a resistance R by a current I is given by the relation $P = I^2R$. To check this relation a student sends several different currents through an unknown resistance immersed in an oil bath and measures the power delivered by measuring the rise in temperature of oil. The results for current (I) and power (P) are given below:

Current I (amp) with negligible uncertainity	Power P (watt) with ± 50
1.5	270
2.0	380
2.5	620
3.0	830
3.5	1280
4.0	1600

Make a plot of P against I^2 including error bars and decide if this experiment is consistent with expected proportionality of P and I^2. Present the best estimate for the value of R. (you may use the method of least squares) [10]

9 + 0.3 + 0.8