Values of some useful constants

Specific heat capacity of ice = 2.10 kJ kg^{-1} K^{-1}

Specific heat capacity of water = 4.20 kJ kg^{-1} K^{-1}

Specific latent heat of fusion of ice = 334 kJ kg^{-1}

Specific latent heat of vaporization of water = 2260 kJ kg^{-1}

Mass of the earth = 5.97×10^{24} kg

Gravitational constant=6.67×10^{-11} N m^2 kg^{-2}

Acceleration due to gravity= 9.81 m s^{-2}

Planck's constant = 6.63×10^{-34} J s

Charge of an electron = 1.60×10^{-19} C

Speed of light in vacuum=3×10^8 m s^{-1}

Permittivity of free space=8.85×10^{-12} C^2 N^{-1} m^{-2}

Stefan's constant=5.67×10^{-8} W m^{-2} K^{-4}

9.1×10^{-31} kg = $m_{electron}$

NePhO

National Physics Olympiad Test- 2017

F.M 100

Time: 3hrs

Attempt all questions

Q1. A pendulum consists of a copper sphere of radius r and density ρ suspended from a string. The motion of the sphere experiences a viscous drag from the air such that the amplitude of oscillation A decays with time t as follows:

$$A = A_o \exp(-\alpha t), \quad \text{where } \alpha = \frac{9\eta}{4\rho r^2},$$

A_o is the amplitude at time t=0 and η is the viscosity of the air. The measurement of the amplitude is accurate to 1%, other measurements are recorded below:

$\eta = (1.78 \pm 0.02) \times 10^{-5}$ kg m^{-1} s^{-1},

r= (5.2 ± 0.1) mm,

$\rho = (8.92 \pm 0.05) \times 10^{3}$ kg m^{-3}.

i) Calculate the percentage error in each of the following: a) η; b) ρ; c) r^2; d) $\frac{A_o}{A}$.

ii) Estimate the time t taken for the amplitude to fall to 85% of A_o. Also calculate the percentage error in $\ln\left(\frac{A_o}{A}\right)$ and t.

iii) Which experimental parameter contributes the largest error to the final result? [10]

Q2. The mean time to failure (t_{MF}) of an integrated circuit is known to obey a law of the form:

$$t_{MF} = C \exp\left(\frac{T_o}{T}\right),$$

where T is the operating temperature and C and T_o are constants. Following values of t_{FM} at various temperatures were obtained from accelerated life-time test.

T(in kelvin)	600	580	560	540	520	500
t_{MF} (in hours)	54	105	206	411	941	2145

i) How will you process the above data to obtain a straight line graph?

ii) Present a table of processed data.

iii) Plot the processed data and fit to a straight line.

iv) Estimate the value of C and T_o from the graph.

v) Calculate the maximum allowable temperature for t_{FM}= 10 years. [You may use the values of C and T_o obtained from the graph.] [10]

Q3. 0.2 kg of steam at 100°C is added to 1 kg of ice at −10°C, keeping the whole system thermally insulated. State the result of mixing and calculate the change in entropy when thermal equilibrium is reached. [10]

Q4. a) i) How far from grains of red sand must you be to position yourself just at the limit of resolving the grain if your pupil diameter is 1.8 mm, the grains are spherical with radius 50 μm, and the light from the grains has wavelength 650 nm?

ii) If the grains were blue and the light from them had wavelength 400 nm, would the answer to (i) be larger or smaller? [5]

b) The Russian physicist P. A. Cerenkov discovered that a charged particle traveling in a solid with the speed of light in that material radiates electromagnetic radiation. What is the minimum kinetic energy in eV that an electron must have while traveling inside a slab of crown glass of refractive index 1.52 in order to create Cerenkov radiation? [5]

Q5. a) A small metal sphere, carrying a net charge of q_1=−2.80 μC is held fixed at certain point on an insulated planar horizontal surface. A second metal sphere with a net charge q_2 = −7.80 μC and mass 1.50 g moves towards q_1. The surface is frictionless. When the two spheres are 0.800 m apart q_2 is moving towards q_1 with the speed of 22 ms^{-1}.

i) What is the speed of q_2 when the spheres are 0.400 m apart?

ii) How close does q_2 get to q_1? [5]

b) An oscillating LC circuit consists of 75.0 mH inductor and 3.60 μF capacitor. If the maximum charge on capacitor is 5.00 μC, what are

i) the total energy in the circuit,

ii) the maximum current,

iii) the period of oscillation? [5]

Q6. a) A current I flows through a solid cylindrical conductor of radius R and uniform resistivity ρ. Find the distribution of current along the radius of the conductor. On the basis of the above analysis, give reasons for the Nepal Electricity Authority laying its conductor with steel core. [5]

b) A car battery with 12 V emf and an internal resistance 0.030 Ω is being charged with a current of 40 A. What are

i) the potential difference across the terminals,
ii) the rate of energy dissipation inside the battery,
iii) the rate of energy conversion to chemical form?

When the battery is used to supply 40 A to the starter motor, what are

iv) the potential difference across the terminals,
v) the rate of energy dissipation inside the battery? [5]

Q7. A chlorine molecule with an initial velocity of 600 ms^{-1} absorbs a photon of wavelength 350 nm and is then dissociated into two chlorine atoms. One of the atoms is detected moving perpendicular to the initial direction of the molecule and having velocity of 1600 ms^{-1}. Neglect the momentum of the absorbed photon. Sketch an appropriate diagram for the situation.
Given the relative atomic mass of chlorine is 35, and the mass of hydrogen atom is 1.67×10^{-27} kg:

i) Calculate the momentum of each chlorine atom.
ii) Is it justified to neglect the momentum of photon?
iii) Calculate the binding energy of the molecule in eV. [10]

Q8. a) A particle of mass m moves in a one-dimensional potential

$$V(x) = A\,|x|,$$

where A is a positive constant.

Use the Heisenberg uncertainty principle to estimate the minimum total energy (kinetic and potential) of the particle as a function of m, A and ħ. [5]

b) Muons are unstable particles; each decays into an electron, a neutrino and an antineutrino. If the number of muons at t=0 is N_o, the number N at time t is given by the expression

$$N=N_o \exp(-t/\tau),$$

where τ=2.20 μs is the mean life time of muon. Suppose the muons move at speed of 0.95 c

i) What is the observed life time of the muons?
ii) How many muons, as a fraction of N_o, remain after traveling a distance of 3.0 km? [5]

Q9. Consider a black hole of mass m.

A black hole is a terrestrial body, which does not allow anything – even light – to escape from inside. The boundary defining a black hole is called event horizon. Only events outside the event horizon can be observed. Thus mass of a black hole and area of the event horizon can only be observed. In absence of any knowledge of anything inside a black hole, we shall use dimensional analysis to gain insight into a black hole.

a) Use dimensional analysis and relate the area of the black hole A with its mass and relevant fundamental constants gravitational constant G and speed of light c. Omitting a numerical factor, we can write

$$A = G^{\alpha} c^{\beta} m^{\gamma}.$$

Find α, β and γ.

b) I) Use the thermodynamic definition of entropy to find the dimensions of entropy. Recall that in thermodynamics temperature is considered a fundamental quantity.

Ii) Assume the entropy of a black hole increases with time. The results from part (a) show that area of the black hole increases with its mass. Therefore, one can argue that entropy is proportional to the area of the event horizon:

$S=\eta A$ (Bekenstein hypothesis), where η is a dimensional constant.

Present dimensional analysis using L, M, T and θ (temperature) as fundamental quantities, η being a constant will depend only on fundamental constants: h (Planck's constant), c, G and k (Boltzmann constant). Taking

$$\eta = h^{a} c^{b} G^{c} k^{d},$$

find a, b, c and d.

c) We wish to understand the evaporation of a black hole, which happens at Hawking temperature and called Hawking radiation.

i) Assuming that the black hole does not do any work on its surrounding, obtain an expression for the Hawking temperature θ_H of the black hole as a function of its mass. [You may use Einstein's mass-energy relation and the first law of thermodynamics.]

ii) Since the black hole is at a temperature θ_H, it will radiate energy according to Stefan-Boltzmann law. Write an expression for the rate of change of energy per unit area $\frac{dE}{dt}$ of the black hole as a function of θ_H.

iii) Obtain an expression for the rate of change of mass of the evaporating black hole ($\frac{dm}{dt}$) in terms of constants and as a function of mass m. From this expression what you can say about the rate of evaporation of black hole as a function of its mass?

iv) Integrate the above equation obtained in (iii) to find the life-time of the black hole.

v) Estimate the life-time of a black hole of earth's mass. [20]