When it comes to experimental physics, graphing skills are one of the most important. So, both International Physics Olympiad (IPhO) and Nepal Physics Olympiad (NePhO) give much emphasis on it.

In NePhO, questions requiring graphing skills carry at least 10 marks (out of 100!). So, if you are preparing for NePhO, graphs are definitely the things you should be familiar with. And if we see the trend of past few years, the questions are always based on one particular skill: linearization. What is it? Let's see with an example asked in NePhO pre-selection 2017.

Question from Nepal Physics Olympiad pre-selections 2017

The voltage V across a charged capacitor is found to decay with time according to the relation

$$V = \alpha e^{-t/T}$$

Where α and T are constants.

i. How would you modify the relation to obtain a straight line for the dependence of V on t?

The table below gives an experimental data for V and t

T/sec	0.0	1.0	2.0	3.0	4.0	5.0	6.0
V/volt	4.3	2.8	1.8	1.2	0.8	0.5	0.3

ii. Copy the table and include the processed data to obtain a single line graph.
iii. Plot the processed data and fit to a straight line.
iv. Estimate the value of α and T from your graph.

Answer

i. The given equation is

$$V = \alpha e^{-t/T}$$

Here, the variables are V and t. And in order to make a straight line out of the equation, we have to convert the equation into the equation of straight line with variables (x and y) depending on V and t. Since the equation is exponential, the easiest way to do so is to take the logarithm on both sides so that the 't' in power descends to the base.
So, taking natural logarithm on both the sides,

$$\ln(V) = \ln(\alpha) - \frac{t}{T}$$

$$\ln(V) = \left(-\frac{1}{T}\right)t + \ln(\alpha)$$

If you compare this equation with y = mx + c,

y = ln(V)

x = t

slope (m) = $\left(-\frac{1}{T}\right)$

Y-intercept(c) = ln(α)

So, if we plot the graph of ln(V) against t, we get a straight line.

If a relationship involves only multiplication and division, (including powers), then logarithms can be used to linearize. Sometimes taking roots or powers of both sides of an equation will help.

If you have another equation that looks like $\mu = a + \frac{b}{\lambda^2}$, and you plot the graph of just μ and λ^{-2}, you have a straight line- the easiest way to do it.

Just analyze the equation.

Your final equation after the linearization should be comparable to the standard equation of straight line

y = mx + c

And on comparison of the equation and the graph, you will get the unknown quantities of the graph.

ii. To obtain the straight line in graph, the values of ln(V) and t are needed. The question provies only V and t. So, we have to process ln(V).

The processed data is as follows:

V (Volt)	t (sec)	ln(V)
4.3	0.0	1.45
2.8	1.0	1.03
1.8	2.0	0.59
1.2	3.0	0.18
0.8	4.0	-0.22
0.5	5.0	-0.69
0.3	6.0	-1.20

The processing of the data involves finding the value of the quantities which are required for plotting the linearized equation. In this case, t and V are already given, but the equation requires ln(V). So, we have to process the quantity.

In another equation taken in the side-note above, you are given μ and λ, but the linearized equation requires μ and λ^{-2}. So, you have to process λ^{-2}.

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iii. For the graph,

Scale:

On X-axis:

7.0 s = 16 units

Here, the most suitable and convenient scale would be to use 2 units as 1.0 s.

So, 2 units = 1.0 s for X-axis

On Y-axis:

20 units = 2.80

1 unit = 0.14 on Y-axis

The points you have plotted should not be congested. Spread them so that your graph cover more than 80% of the paper if possible.

And for this, taking the scale of the graph needs some thought. Unitary method should be useful. For this, always take some units (of your variable) greater than the actual spread (like I took time 7.0 s even though the question requires only 6.0 s) so that you can extend your graph a bit beyond the given points. Equate your variable's unit with the total units (squares) in graph. In this case you will get something like 1.0 s = 2.29 units. But that is bit difficult to plot. So, look for the easiest units. In this case, if I take just 14 units, I will get 1.0 s = 2 units, which is far easier and I do not lose much of the space too.

Similarly, for another axis, I have a total of 22 units (squares), but it would be far easier if I took just 20 units.

So, have some thoughts while taking our scale.

Points to be taken care while doing the graph:

a) The graph should cover over 80% of the space of the paper.
b) You should write the scale used in the graph (usually written at the top or bottom of paper)
c) You should indicate what quantity the X and Y-axis represent. If any quantity in expressed in scientific notation (like 2.3×10^3), you can just put 2.3 in the graph and you could write ($\times 10^3$) along with the quantity you have indicated along the axes. *Like in this graph, if the times were* 1.0×10^2, 2.0×10^2 *and so on, you could have written 1.0, 2.0… on the axes, and 'time (*$\times 10^2$ *s)' in the plate where there is just 'time (in s)' now.*
d) *Your points in the graph should be easily distinguishable. You can draw a circle around the plotted points for this as shown in the graph below.*
e) While drawing the line, a single line joining all the points might not be obtained. In this case, you have to draw the line of best fit by drawing a line such that equal proportion of points lie above and below it

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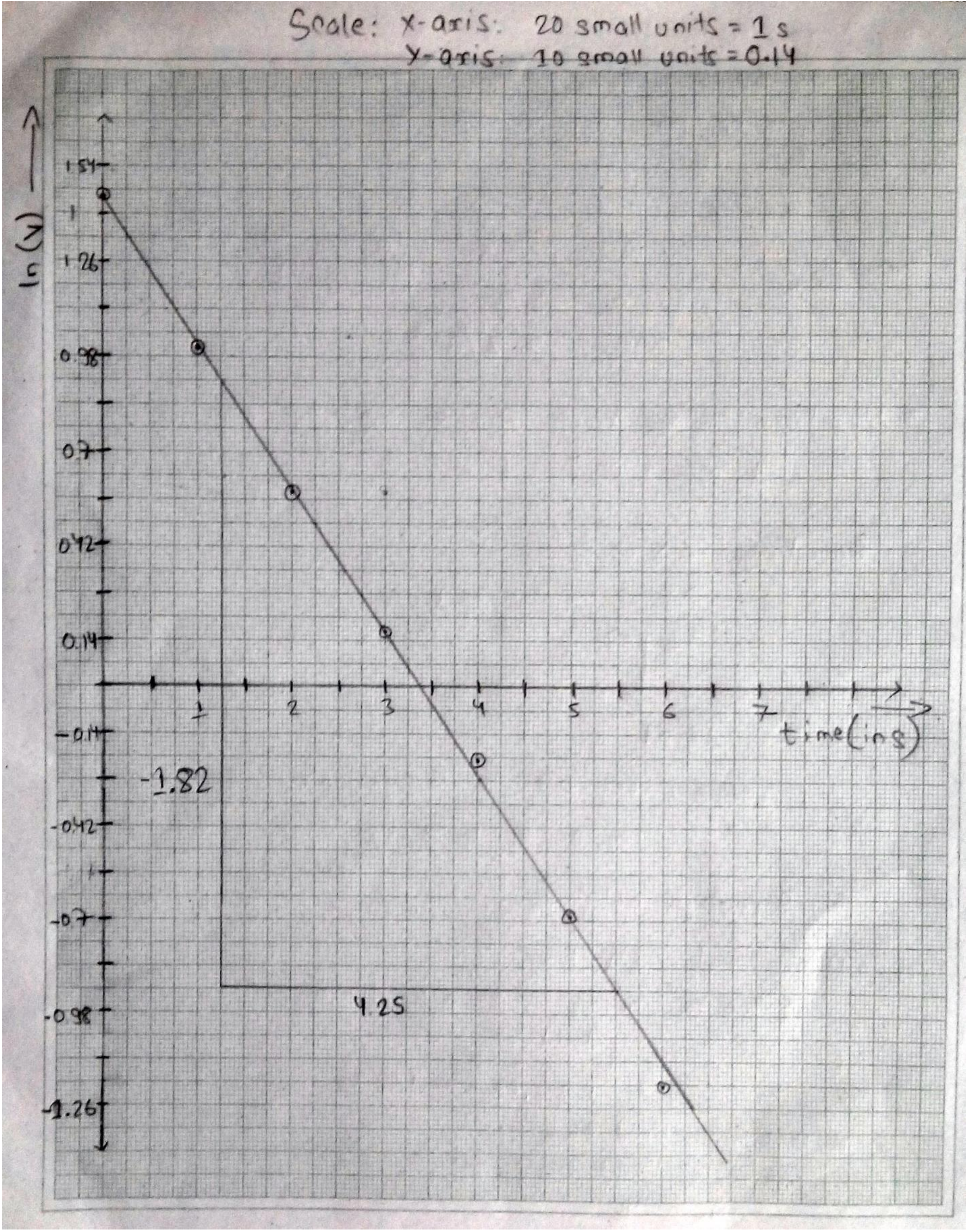

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iv. From the graph,

Slope = $\frac{-1.82}{4.25} = -0.43$

Since slope(m) = $\left(-\frac{1}{T}\right)$

Or, $\left(-\frac{1}{T}\right) = -0.43$

So, T = 2.33

Also,

Y-intercept = 1.46

Or, ln(α) = 1.46

So, α = 4.3

To find the slope, take any two points on the straight line, and draw a right angled triangle with the portion of the line joining the two points as the hypotenuse. The farther the points and the larger the triangle, the better it is for you as well as for the one who checks it.