E= mc^2 ρhysics Ωlympiad

Symbol No.: ………………………………………………………

College: ………………………………………………………

Grade:…………

Some physical constants:

g = 9.8 N/kg

G = 6.67 × 10−11 N · m^2/kg^2

k = 1/4πε0 = 8.99 × 109 N · m^2/C^2

c = 3.00 × 108 m/s

k_B = 1.38 × 10−23 J/K

N_A = 6.02 × 1023 $(mol)^{-1}$

R = $N_A k_B$ = 8.31 J/(mol · K)

σ = 5.67 × 10−8 J/(s · m^2 · K^4)

e = 1.602 × 10−19 C

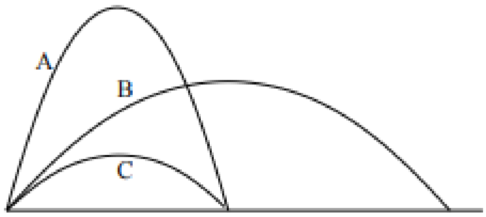
1eV = 1.602 × 10−19 J

Note: ***The questions might look harder than they actually are.***

MCQ's

1. The speed of light in a vacuum, c, depends on two fundamental constants; the permeability of free space, μ_0, and the permittivity of free space, ε_0. The speed of light is related to these constants by $c = (\mu_0 \varepsilon_0)^{-1/2}$. The SI units of ε_0 are $N^{-1} C^2 m^{-2}$ and c are ms^{-1}, where N is Newton and C is Coulomb. The units of μ_0 are
   a. $kg^{-1} m^{-1} C^2$
   b. $kg^{-1} s^{-3} C^{-2}$
   c. $kg\ m\ C^{-2}$
   d. It is dimensionless

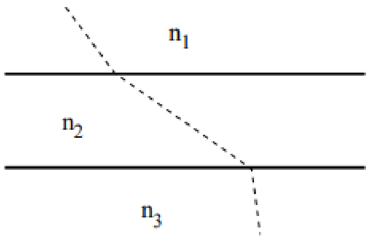
2. The diagram shows the trajectory of three artillery shells. Each was fired with the same initial speed. Which shell was in the air for the longest time? (Ignore air friction.)



   a. Shell A.
   b. Shell B.
   c. Shell C.
   d. Shells A and C were in the air for equal time, which was longer than for shell B.

3. A fat tree frog jumps from a tree in an attempt to catch a small fly. Both are in mid–air and have the same kinetic energy. Which of the following statements is true?
   a. The fly has a greater speed than the tree frog.
   b. The tree frog has a greater speed than the fly.
   c. The fly and the tree frog have the same speed.
   d. The kinetic energy cannot give information about their speeds.

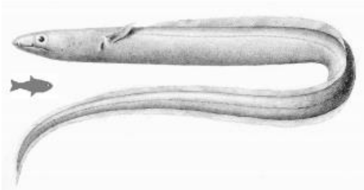
4. The figure shows the path of a ray of light as it passes through three different materials with refractive indices n1, n2 and n3. The figure is drawn to scale. What can we conclude concerning the indices of these three materials?



   a. n3 < n2 < n1.
   b. n3 < n1 < n2.
   c. n2 < n1 < n3.
   d. n1 < n3 < n2.

5. When objects at different distances are seen by a normal eye, which of the following remains constant?
   a. The focal length of the eye-lens,
   b. The object distance from the eye-lens,
   c. The radius of curvature of the eye-lens,
   d. The image distance from the eye-lens.

6. An electric eel is capable of producing an 860 volt difference between its head and tail. When using this ability to stun prey, the electric eel will sometimes form a U shape as shown in Figure alongside. This has the effect that the eel's prey is more likely to be stunned. Why is this?



   a. The voltage difference between head and tail is increased.
   b. The electric field between the head and tail is increased.
   c. The electric field near the center of the eel is increased.
   d. The voltage at the head and tail are increased by the same amount.

7. Three identical resistors are connected across a voltage source V so that one of them is in parallel with two others which are connected in series. The power dissipated through the first one, compared to the power dissipated by each of the other two, is approximately
   a. the same.
   b. half as much.
   c. twice as much.
   d. four times as much.

Short Questions

1. If you dive into water, you reach greater depth than if you jump flat. Explain this difference in depth.

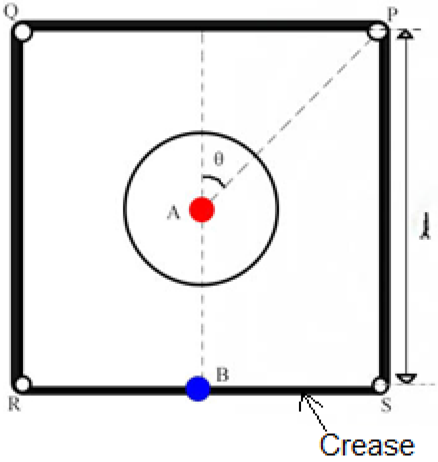
2. An ice block (density: 910 kg/m^3) of volume 1 m^3 floats on the surface of salt water (density: 1030 kg/m^3) in a cylindrical vessel of base area 1 m^2. When the ice melts down, what is the change in the level of liquid in the vessel?

3. To withstand the harsh weather of the Antarctic, emperor penguins huddle in groups. Assume that a penguin is a circular cylinder with a top surface area A= 0.34 m^2 and height H= 1.1 m. Let P_r be the rate at which an individual penguin radiates energy to the environment (through the top and the sides); thus NP_r is the rate at which N identical, well-separated penguins radiate. If the penguins huddle closely to form a huddled cylinder with top surface area Na and height h, the cylinder radiates at the rate P_h. If N= 1000, (a) what is the value of the fraction P_h/NP_r and (b) by what percentage does huddling reduce the total radiation loss?

4. A ring with a colorless gemstone is dropped into water. The gemstone becomes invisible when submerged. Can it be diamond?

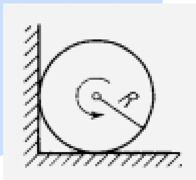
Numericals

1. **Pocket!:** A carom coin A of mass m, which is at the center of a square carom board of length l, is to be hit with the striker coin B of mass M=3m. Assume the crease for the striker be at one of the side of the board and the striker is in the middle of the side, as in figure. No powder has been used on the board due to which the coins slow down on sliding.



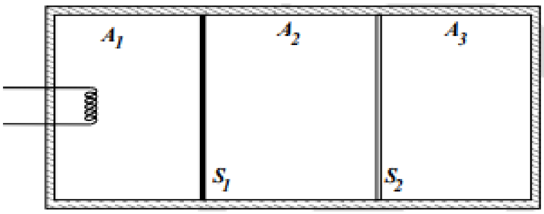
   I. You win the game only when you *score* the coin in the pocket P. At what angle (θ) (relative to the line joining coins A and B) should the coin A move for the score?
   II. Assuming the coefficient of friction between the coins and the board to be μ, find the minimum velocity ($v_{A\text{-}min}$) with which coin A should leave the center after being hit by the striker so that it just enters the pocket.
   III. Assuming the collision between the coins to be perfectly elastic, and the coins to be point masses, find the minimum velocity ($v_{B\text{-}min}$) with which the striker B should be released for scoring coin A.
   IV. To strike the coin B be finger, assume that the time of impact between the finger and coin is t. Find the minimum force (F_{min}) to be applied by the finger to the coin.
   V. Given that m = 15 gm, $\mu = \frac{1}{\sqrt{2}}$, l = 75 cm, and t = 0.1 s, calculate F_{min}.

2. This question has two distinct parts.
   I. A uniform cylinder of radius R is spinned about its axis to the angular velocity ω_0 and then placed into a corner. The coefficient of friction between the corner walls and the cylinder is equal to k. How many turns will the cylinder accomplish before it stops?



   II. A 5m long cylindrical steel wire with radius 2 x 10-3m is suspended vertically from a rigid support and carries a bob of mass 100kg at the other end. If the bob gets snapped; calculate the change in temperature of the wire ignoring radiation losses. (For the steel wire: Young's Modulus = 2.1 x 10^11 Pa; Density = 7860 kg/m^3; Specific heat = 420 J/kg-K).

3. Consider a closed cylinder whose walls are adiabatic. The cylinder lies in a horizontal position and is divided into three parts A_1, A_2 and A_3 by means of partitions S_1 and S_2 which can move along the length of the cylinder without friction. S_1 is adiabatic while S_2 is conducting. Initially each of the parts A_1, A_2 and A_3 contains one mole of Helium gas at pressure P_0, temperature T_0, and volume V_0.



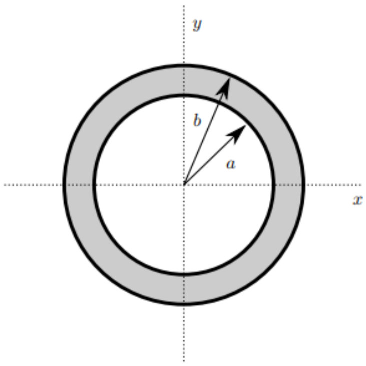
Helium is to be regarded as an ideal monoatomic gas. Also $C_v = 3R/2$, $C_p = 5R/2$ for Helium. Now heat is slowly supplied to the gas in part A1 till the temperature in part A3 becomes $T_3 = 9T_0/4$.

I. Let the final thermodynamic variables of the partitions A1, A2, and A3 be $\{P_1, V_1, T_1\}$, $\{P_2, V_2, T_2\}$ and $\{P_3, V_3, T_3\}$ respectively. Express the pressure in terms of P_0, volume in terms of V_0, and temperature in terms of T_0.

II. What work is done by the gas in A_1 on the gases in A_2 and A_3 in terms of $\{P_0, V_0\}$ and related quantities?

4. This question has two distinct parts.

I. A positive point charge q is located inside a neutral hollow spherical conducting shell. The shell has inner radius a and outer radius b; b − a is not negligible. The shell is centered on the origin.



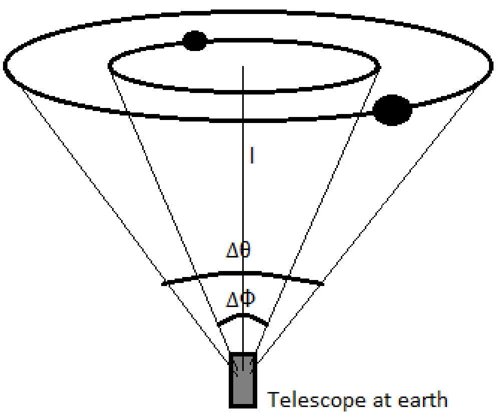
   i. Determine the magnitude of the electric field outside the conducting shell at x = b.
   ii. Sketch a graph for the magnitude of the electric field versus the distance from the origin along x-axis.
   iii. Determine the electric potential at x = a.
   iv. Sketch a graph for the electric potential along the x axis versus the distance from the origin along x-axis.

II. A conducting sphere of radius r is at an electric potential V. Show that the surface charge density is inversely proportional to the radius of the sphere. How does this explain the corona discharge phenomenon?

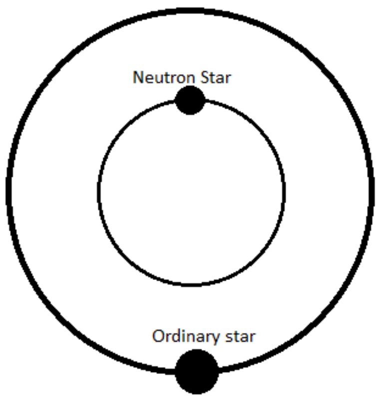
Long Application Type:

Binary Stars

Two stars rotating around their common center of mass with equal angular speed form a binary star system. Almost half of the stars in our galaxy are binary star systems.



One type of binary system consists of an ordinary star with mass m_0 and radius R, and a more massive (in mass), compact neutron star with mass M, rotating around each



Top view of the system

other in circular orbits. In all the following, ignore the motion of the earth.

Observations of such a binary system reveal the following information:

- Two diametrically opposite points on the orbit of ordinary star subtend an angle of $\Delta\theta$, while that in orbit of neutron star is $\Delta\Phi$.(see Fig. 1).
- The time they take to reach from a point to its diametrically opposite point is τ.
- The surface temperature of ordinary star is measured to be T, and the system is at a distance *l* from the earth

Questions:

a. Express the radii of orbit of the normal star r1 and neutron star r2 in terms of l. (As the star is very far from earth, l >> r1 and l >> r2.)

b. When the stars complete one rotation about their common center of mass, we observe two maximum displacements.
What is their angular speed (w)?

c. The gravitational force between the stars provides them the necessary centripetal force to rotate about their common center of mass. For both stars, write the expression for it.
Also, find the mass m0 of the normal star in terms of r1, r2 and w.
Using the results from a and b, rewrite the equation to obtain m0 in terms of l.

d. Assuming that the normal star behaves as a perfect black body, find the total energy it radiates per second.

This energy is distributed uniformly over a larger sphere as the radiations spread away from the star. Find the radiated energy P incident on a unit area on earth's surface per unit time.
Most of the stars generate energy through the same mechanism. Because of this, there is an empirical relation between their mass, M , and their radius, R. This relation could be written in the form:

$$(R/R_{sun}) = (M/M_{sun})^{\alpha}$$

Here, $M_{sun} = 2.0 \times 10^{30}$ kg is the solar mass and $R_{sun} = 7.0 \times 10^{8}$ m is the radius of sun.

e. In order to find the value of α, we have to linearize the equation first. So, take natural logarithm on both sides of the equation.

f. Process the data below to obtain a straight line.

(M/M_{sun})	2.1	3.2	6.5	18.0	40.0
(R/R_{sun})	1.3	2.5	5.5	7.4	16.6

g. Plot the graph and fit to a straight line. What is the value of α?

h. If the mass of a star is 1.7 times that of the sun, find its radius.